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MARKOVIAN INTEGRATED INVENTORY MODEL FOR DETERIORATING ITEMS WITH BAYESIAN ESTIMATION

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Abstract

A Markovian inventory integrated policy with inter-demand time as exponential distribution is considered in this paper. This paper presents, a single-vendor and multiple-buyers integrated inventory model for deteriorating items. The model contains the exponential parameter which is unknown and is estimated through MLE and Baye's under a squared error loss function. The conjugate Gamma prior is used as the prior distribution of exponential distribution. Finally, a numerical MCMC simulation is used to compare the estimators obtained with Expected risk and are shown graphically. The proposed model is formulated and the optimal decisions are obtained by minimising the average total cost of the integrated system. A numerical example is taken to illustrate the developed model. The sensitivity analysis is also carried out for the model with percentage change in the parameters.

Keywords:

Bayesian Estimation;
Deterioration;
Integrated Inventory policy;
Single-vendor and Multiple buyers;
Stochastic demand.

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1. Introduction

Most of the inventory models in the existing literature assume that a single entity, i.e. a buyer or a vendor manages its inventory by minimizing the total cost or maximizing the total profit of the system. The individual decisions made by the buyer or the vendor cannot assure that the two parties as a whole would reach the optimal state. In the global competitive market today, the buyer and the vendor should not be considered as just two separate entities; they should be treated as strategic partners with a long-term relationship. A buyer has the privilege of deciding the ordering cycle in every competitive situation. But the optimal ordering cycle favoured by the buyer may not be the most economical for the vendor.

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The single-vendor multiple-buyers integrated inventory system received a lot of attention in recent years. This renewed interest is motivated by the growing focus on supply chain. Firms are realizing that a more efficient management of inventories across the entire supply chain through better coordination and more cooperation is in the joint benefit of all parties involved. Due to rising costs, shrinking resources, shortened product life cycle, increasing competitive pressure and quicker response, much attention has been paid tothe collaboration between the members of the supply chain The collaboration involves commitment whichcreates a beneficial environment for them. While maintaining the inventories, deterioration of items is inherent. Most of the physical goods deteriorate over time. Deterioration defined as decay, damage, spoilage, evaporation, obsolescence, pilferage and loss of entity or loss of marginal value of a commodity that the results in decreasing usefulness from the original one. The control and maintenance of inventories for deteriorating items have received great attention in recent years. Many of the researchers in deteriorating inventory have assumed constant rate of deterioration as well as the function of time.

Several researchers have studied the integrated buyer-vendor inventory problems which are available in the literature. Goyal S. K and Gupta Y.P (1989) have reviewed the buyer vendor coordination from integrated inventory models. P. C. Yang and H. M. Wee (2000) focus on the economic ordering policy of deteriorated items for single vendor and single buyer inventory problem. Woo Y.Y et. al., (2001) deals with an ordering cost reduction integrated inventory model for single vendor and multiple buyers. Integrated inventory model for deteriorating items under a multi-echelon supply chain environment is established by Rau. Wu, et.al (2003). Chang H.C., et.al. (2006) have discussed an integrated vendor-buyer co-operative inventory models with controllable lead time and ordering cost reduction. Chung K. J., (2008) was given an Improvement of an integrated single-vendor single-buyer inventory model with shortages. Hoque M. A., (2008) has proposed synchronization in the single- manufacturer multi-buyer integrated inventory supply chain. Debashish Kumar et. al., (2017) have presented a multi-vendor single-buyer integrated inventory model with shortages.

In most of the research papers in the literature assumes that the demand is deterministic, whereas in this model the inter-demand time is assumed to be exponential distribution. The parameter involved in the model is assumed to be unknown and is estimated through Baye's and MLE. One of the best systematic methods for incorporating current demand information is known to be the Bayesian approach. The Bayesian approach can be applied to inventory system with either a finite or an infinite planning horizon. Items like computers and related products or even motor vehicles, are being generally have finite planning horizons with fluctuating demands, and Bayesian set-up could be appropriate. The Bayesian approach can also applied to infinite horizon problems in the initial stages, until the demand for the product stabilizes or enough data accumulates for using other estimation procedures. For this purpose, a prior distribution is considered for the unknown parameter for Baye's estimation.

In this paper presents, a single-vendor and multiple-buyers integrated inventory model for deteriorating items is developed. The objective of the paper is to develop an optimum policy that minimizes the total average cost by using the above estimates of the parameter. A special case for single vendor and two buyers is modelled and for estimating the parameter, the conjugate Gamma prior is used as the prior distribution of exponential distribution. The model is also illustrated numerically and a numerical MCMC simulation is used to compare the estimators of Baye's and MLE which are shown also graphically .The sensitivity analysis is also carried out for the model with percentage change in the parameter.

2. Assumptions and Notations

The Mathematical model is under the following assumptions

1. An Inventory system of single vendor and N buyers is considered
2. The inventory system deals with a single item.
3. The demand decreasing function of time t.

4. Shortages of an item at any stage are not allowed.
5. The replenishment rate is instantaneous. i.e., lead time is zero.
6. The deterioration units can neither be repaired nor replaced during the cycle time.

The proposed model is derived using the following notations

N : Number of buyers in the system
λ_i (t) : Exponential rate of demand with respect to time for the i^{th} buyer
T : Vendor's cycle time (decision variable)
n_i : Number of orders during cycle time T for the i^{th} buyer (decision variable).
θ : Deterioration rate ($0 < \theta < 1$) per unit time
$I_v(t)$: Inventory level for vendor's at any time t, $0 < t < T$
$I_{bi}(t)$: Inventory level for i^{th} buyer at any time t , $0 < t < \frac{T}{n_i}$
I_{mv} : The maximum inventory level of vendor.
I_{mbi} : The maximum inventory level of i^{th} buyer.
I_v : Vendor's inventory carrying cost per unit time.
I_b : Buyer's inventory carrying cost per unit time.
DC_v : Vendor's deteriorating cost per unit time.
DC_b : Buyer's deteriorating cost per unit time.
C_v : Vendor's purchasing cost per unit time.
C_b : Buyer's purchasing cost per unit time.
K_v : Total cost of vendor per unit time.
K_b : Total cost of buyer's per unit time.
K : Integrated total cost of vendor and buyer.

3. Description of the Model

Let $I_{bi}(t)$ is inventory level for buyer i (i = 1,2...,N) at any instant of time, $0 < t < \frac{T}{n_i}$ and let $I_v(t)$ is inventory level for vendor at any instant of time, $0 < t < T$. The inventor level depletes due to demand and deterioration of items. The differential equation for vendor and buyer's are given by

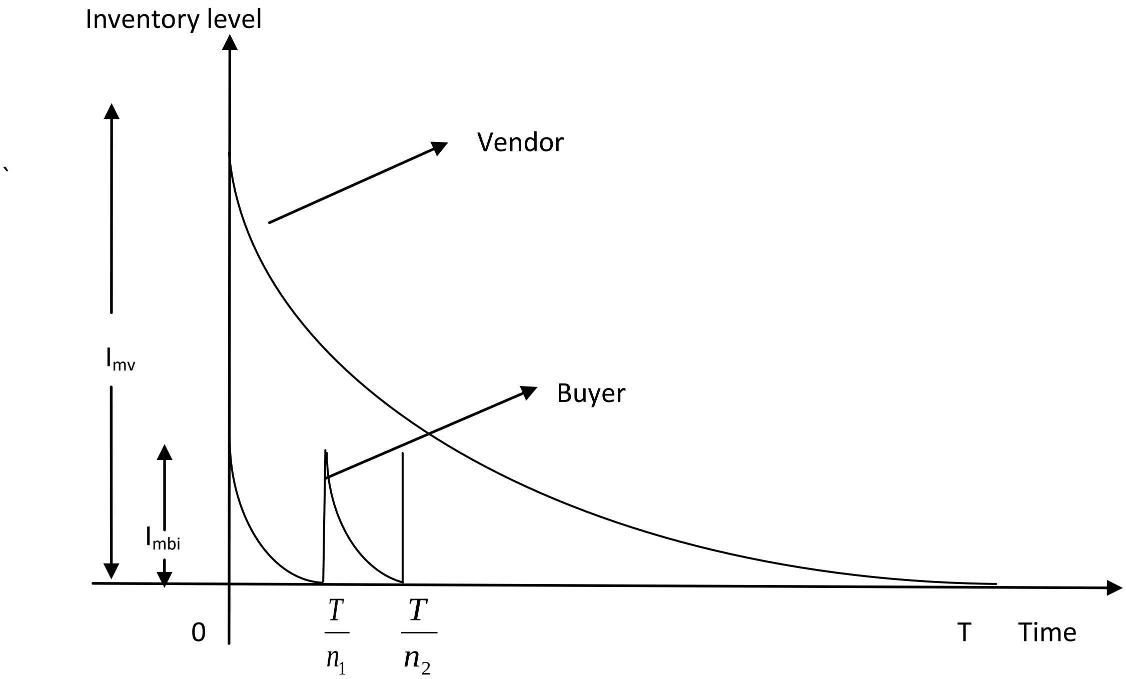

Figure 1 Vendor – Buyers inventory system

$$\frac{d\,I_v(t)}{dt} + \theta I_{vi}(t) = \sum_{i=1}^{n} \lambda_i e^{-\lambda i t} \quad , \quad 0 < t < T \quad ----(1)$$

$$\frac{d\,I_{bi}(t)}{dt} + \theta I_{bi}(t) = -\lambda_i e^{-\lambda i t} \quad , \quad 0 < t < \frac{T}{n_i} \quad i = 1,2 \ldots N \quad ----(2)$$

Using various boundary conditions $I_v(t) = 0$, $I_{bi}\left(\frac{T}{n_i}\right) = 0$, the solutions of the above differential equations are

$$I_v(t) = \sum_{i=1}^{N} \frac{\lambda_i e^{-\lambda i t}}{(\lambda_i - \theta)} - \sum_{i=1}^{N} \frac{\lambda_i e^{-T(\lambda_i - \theta)}}{(\lambda_i - \theta)} e^{-\theta t} \quad 0 < t < T \quad ----(3)$$

$$I_{bi}(t) = \frac{\lambda_i e^{-\lambda i t}}{(\lambda_i - \theta)} - \frac{\lambda_i e^{-\frac{T}{ni}(\lambda i - \theta)}}{(\lambda_i - \theta)} e^{-\theta t} \quad 0 < t < \frac{T}{n_i} \quad i = 1,2 \ldots N \quad ----(4)$$

Using $I_v(0) = I_{mv}$, $I_{bi}(0) = I_{mbi}$ the maximum inventory for vendor and buyers are

$$I_{mv} = \sum_{i=1}^{N} \frac{\lambda_i}{(\lambda_i - \theta)} - \sum_{i=1}^{N} \frac{\lambda_i e^{-T(\lambda_i - \theta)}}{(\lambda_i - \theta)} \quad ----(5)$$

$$I_{mbi} = \frac{\lambda_i}{(\lambda_i - \theta)} - \frac{\lambda_i e^{-\frac{T}{ni}(\lambda i - \theta)}}{(\lambda_i - \theta)} \quad i = 1,2 \ldots N \quad ----(6)$$

During [0, T] inventory level is $\int_0^{\frac{T}{ni}} I_{bi}(t)\,dt$ for buyer i. Therefore, average inventory level for all the buyers per unit time is $\frac{1}{T}\sum_{i=1}^{N} n_i \int_0^{\frac{T}{n_i}} I_{bi}(t)\,dt$. While vendor's average inventory level per unit time is $\frac{1}{T}\int_0^{T} I_v(t)\,dt$. Hence, yearly inventory holding cost for N buyer and the vendor is given by

$$IHC_b = C_b I_b \; \frac{1}{T}\sum_{i=1}^{N} n_i \int_0^{\frac{T}{n_i}} I_{bi}(t)\,dt ----(7)$$ and

$$IHC_v = C_v I_v \left[\frac{1}{T}\int_0^{T} I_v(t)\,dt - \frac{1}{T}\sum_{i=1}^{N} n_i \int_0^{\frac{T}{n_i}} I_{bi}(t)\,dt \right] ----(8)$$

Annual deterioration rate for all buyers and vendor during [0,T] are

$$DC_b = C_b \theta \; \frac{1}{T}\sum_{i=1}^{N} n_i \int_0^{\frac{T}{n_i}} I_{bi}(t)\,dt ----(9)$$ and

$$DC_v = C_v \theta \left[\frac{1}{T}\int_0^{T} I_v(t)\,dt - \frac{1}{T}\sum_{i=1}^{N} n_i \int_0^{\frac{T}{n_i}} I_{bi}(t)\,dt \right] ----(10)$$

Setup cost for all buyers and vendor during [0,T] are

$$OC_b = \frac{1}{T}\sum_{i=1}^{N} n_i A_{bi} ----(11)$$ and

$$OC_v = \frac{A_v}{T} ----(12)$$

Hence, The buyer's Total cost, $K_b = IHC_b + DC_b + OC_b$ $----(13)$

The Vendor's Total cost $K_v = IHC_v + DC_v + OC_v$ $----(14)$

The Integrated Total cost of vendor and N buyers in K is the sum of equations (13) and (14)

$K (T, n_i) = K = K_b + K_v$ $----(15)$

where K is a function of discrete variable n_i and continuous variable T. i=1,2...N

4. Computation procedure

The objective of the model to obtain the value of n_i, which minimizes integrated cost K, where i = 1,2...N. Since the number of delivery n_i, per order cycle T is a discrete variable. The following steps are to derive value of n_i.

Step:1 To derive optimal solution, the necessary condition is $\frac{\partial K}{\partial T} = 0$. For each n_i, denote order cycle T by notation T (n_i), where i = 1,2...N.

Step:2 To find the optimal solution of n_i, such that, the following condition must satisfy:

$$K(n_i^* -1, T(n_i^* -1)) > K(n_i^*, T(n_i^*)) < K(n_i^* +1, T(n_i^* +1))$$

such that $K(n_i^*, T(n_i^*))$ is the optimal value of integrated cost.

5. Special case: Single vendor and two buyers inventory model

Let $I_{b1}(t)$ is inventory level for buyer 1,at any instant of time $0 < t < \frac{T}{n_1}$, $I_{b2}(t)$ is inventory level for buyer 2, at any instant of time $0 < t < \frac{T}{n_2}$ and let $I_v(t)$ is inventory level for vendor at any instant of time, $0 < t < T$. The inventor level depletes due to demand and deterioration of items. The differential equation for vendor and buyer's are given by

$$\frac{d I_v(t)}{dt} + \theta I_{vi}(t) = \sum_{i=1}^{2} \lambda_i e^{-\lambda it} \quad , \quad 0 < t < T \quad ----(16)$$

$$\frac{d I_{b1}(t)}{dt} + \theta I_{b1}(t) = -\lambda_1 e^{-\lambda_1 t} \quad , \quad 0 < t < \frac{T}{n_1} \quad ----(17)$$

$$\frac{d I_{b2}(t)}{dt} + \theta I_{b2}(t) = -\lambda_2 e^{-\lambda_2 t} \quad , \quad 0 < t < \frac{T}{n_2} \quad ----(18)$$

Using various boundary conditions $I_v(t) = 0$, $I_{b1}\left(\frac{T}{n_1}\right) = 0$, $I_{b2}\left(\frac{T}{n_2}\right) = 0$ the solutions of the above differential equations are

$$I_v(t) = \left[-\lambda_1 t - \frac{\lambda_1 \theta t^2}{2} - \lambda_2 t - \frac{\lambda_2 \theta t^2}{2} + \lambda_1 \theta t^2 + \lambda_2 \theta t^2 \right] + \lambda_1 T + \frac{\lambda_1 \theta T^2}{2}$$

$$+ \lambda_2 T + \frac{\lambda_2 \theta T^2}{2} - \lambda_1 \theta T^2 - \lambda_2 \theta T^2 + \lambda_1 \theta T + \lambda_2 \theta T - \lambda_1 \theta t T - \lambda_2 \theta t T ----(19)$$

$$I_{b1}(t) = -\lambda_1 t - \frac{\lambda_1 \theta t^2}{2} + \lambda_1 \theta t^2 + \frac{\lambda_1 T}{n_1} + \frac{\lambda_1 \theta T^2}{2 n_1^2} - \frac{\lambda_1 \theta T^2}{n_1^2} + \frac{\lambda_1 \theta T^2}{\theta T} - \frac{\lambda_1 \theta t T}{n_1} ----(20)$$

$$I_{b2}(t) = -\lambda_2 t - \frac{\lambda_2 \theta t^2}{2} + \lambda_2 \theta t^2 + \frac{\lambda_2 T}{n_2} + \frac{\lambda_2 \theta T^2}{2 n_2^2} - \frac{\lambda_2 \theta T^2}{n_2^2} + \frac{\lambda_2 \theta T^2}{\theta T} - \frac{\lambda_2 \theta t T}{n_2} ----(21)$$

Using $I_v(0)=I_{mv}$, $I_{b1}(0)=I_{mb1}$, $I_{b2}(0)=I_{mb2}$ the maximum inventory for vendor and buyers are

$$I_{mv}=\lambda_1 T+\frac{\lambda_1\theta T^2}{2}+\lambda_2 T+\frac{\lambda_2\theta T^2}{2}-\lambda_1\theta T^2-\lambda_2\theta T^2+\lambda_1\theta T+\lambda_2\theta T \quad ----(22)$$

$$I_{mb1}=\frac{\lambda_1 T}{n_1}+\frac{\lambda_1\theta T^2}{2n_1^2}-\frac{\lambda_1\theta T^2}{n_1^2}+\frac{\lambda_1\theta T^2}{\theta T} \quad ----(23)$$

$$I_{mb2}=\frac{\lambda_2 T}{n_2}+\frac{\lambda_2\theta T^2}{2n_2^2}-\frac{\lambda_2\theta T^2}{n_2^2}+\frac{\lambda_2\theta T^2}{\theta T} \quad ----(24)$$

During [0,T] inventory level is $\int_0^{\frac{T}{n_1}} I_{b1}(t)\,dt$ for buyer 1, and inventory level is $\int_0^{\frac{T}{n_2}} I_{b2}(t)\,dt$ for buyer 2.

Therefore, average inventory level for both the buyers per unit time is $\frac{n_1+n_2}{T}\left[\int_0^{\frac{T}{n_1}} I_{b1}(t)\,dt+\int_0^{\frac{T}{n_2}} I_{b2}(t)\,dt\right]$. While vendor's average inventory level per unit time is $\frac{1}{T}\int_0^{T} I_v(t)\,dt$. Hence, yearly inventory holding cost for both the buyer's and the vendor is given by

$$IHC_b=C_b\,I_b\,\frac{n_1+n_2}{T}\left[\int_0^{\frac{T}{n_1}} I_{b1}(t)\,dt+\int_0^{\frac{T}{n_2}} I_{b2}(t)\,dt\right]$$

$$IHC_v=C_v\,I_v\left[\frac{1}{T}\int_0^{T} I_v(t)\,dt-\frac{n_1+n_2}{T}\left[\int_0^{\frac{T}{n_1}} I_{b1}(t)\,dt+\int_0^{\frac{T}{n_2}} I_{b2}(t)\,dt\right]\right]$$

Annual deterioration rate for both the buyers and vendor during [0, T] are

$$DC_b=C_b\,\theta\frac{n_1+n_2}{T}\left[\int_0^{\frac{T}{n_1}} I_{b1}(t)\,dt+\int_0^{\frac{T}{n_2}} I_{b2}(t)\,dt\right]$$

$$DC_v=C_v\theta\left[\frac{1}{T}\int_0^{T} I_v(t)\,dt-\frac{n_1+n_2}{T}\left[\int_0^{\frac{T}{n_1}} I_{b1}(t)\,dt+\int_0^{\frac{T}{n_2}} I_{b2}(t)\,dt\right]\right]$$

The Setup cost for both the buyers and vendor during [0, T] are

$$OC_b=\frac{n_1 A_{b1}}{T}+\frac{n_2 A_{b2}}{T} \quad ----(29)$$

$$OC_v=\frac{A_v}{T} \quad ----(30)$$

The Buyer's Total cost, $K_b = IHC_b + DC_b + OC_b$

$$K_b = -\frac{C_b I_b \lambda_1 T}{2n_1} - \frac{C_b I_b \lambda_1 \theta T^2}{6n_1^2} + \frac{C_b I_b \lambda_1 \theta T^2}{3n_1^2} + \frac{C_b I_b \lambda_1 T}{n_1} + \frac{C_b I_b \lambda_1 \theta T^2}{2n_1^2} - \frac{C_b I_b \lambda_1 \theta T^2}{n_1^2}$$
$$+ C_b I_b \lambda_1 T - \frac{C_b I_b \lambda_1 \theta T^2}{2n_1^2} - \frac{C_b I_b \lambda_2 T}{2n_2} - \frac{C_b I_b \lambda_2 \theta T^2}{6n_2^2} + \frac{C_b I_b \lambda_2 \theta T^2}{3n_2^2} + \frac{C_b I_b \lambda_2 T}{n_2}$$
$$+ \frac{C_b I_b \lambda_2 \theta T^2}{2n_2^2} - \frac{C_b I_b \lambda_2 \theta T^2}{n_2^2} + C_b I_b \lambda_2 T - \frac{C_b I_b \lambda_2 \theta T^2}{2n_2^2} - \frac{C_b \theta \lambda_1 T}{2n_1} + C_b \theta \lambda_1 T$$
$$+ \frac{C_b \theta \lambda_1 T}{n_1} - \frac{C_b \theta \lambda_2 T}{2n_2} + C_b \theta \lambda_2 T + \frac{C_b \theta \lambda_2 T}{n_2} + \frac{n_1 A_{b1}}{T} + \frac{n_2 A_{b2}}{T} ----(31)$$

The Vendor's Total cost $K_v = IHC_v + DC_v + OC_v$

$$K_v = C_v I_v \lambda_1 \theta T + C_v I_v \lambda_2 \theta T + \frac{C_v I_v \lambda_1 \theta T^2}{3} + \frac{C_v I_v \lambda_2 \theta T^2}{3} - C_v I_v \lambda_1 \theta T^2 - C_v I_v \lambda_2 \theta T^2$$
$$- \frac{C_v I_v \lambda_1 \theta T^2}{6} - \frac{C_v I_v \lambda_2 \theta T^2}{6} + \frac{C_v I_v \lambda_1 T}{2n_1} + \frac{C_v I_v \lambda_1 \theta T^2}{6n_1^2} - \frac{C_v I_v \lambda_1 \theta T^2}{3n_1^2} - \frac{C_v I_v \lambda_1 T}{n_1}$$
$$- \frac{C_v I_v \lambda_1 \theta T^2}{2n_1^2} + \frac{C_v I_v \lambda_1 \theta T^2}{n_1^2} - C_v I_v \lambda_1 T + \frac{C_v I_v \lambda_1 \theta T^2}{2n_1^2} + \frac{C_v I_v \lambda_2 T}{2n_2} + \frac{C_v I_v \lambda_2 \theta T^2}{6n_2^2}$$
$$- \frac{C_v I_v \lambda_2 \theta T^2}{3n_2^2} - \frac{C_v I_v \lambda_2 T}{n_2} - \frac{C_v I_v \lambda_2 \theta T^2}{2n_2^2} + \frac{C_v I_v \lambda_2 \theta T^2}{n_2^2} - C_v I_v \lambda_2 T + \frac{C_v I_v \lambda_2 \theta T^2}{2n_2^2}$$
$$+ \frac{C_v \theta \lambda_1 T}{2n_1} - \frac{C_v \theta \lambda_1 T}{n_1} - C_v \theta \lambda_1 T + \frac{C_v \theta \lambda_2 T}{2n_2} - \frac{C_v \theta \lambda_2 T}{n_2} - C_v \theta \lambda_2 T + \frac{A_v}{T} ----(32)$$

The Integrated Total cost of vendor and N buyers in K is the sum of equations (31) and (32)
$K(T, n_1 \text{ and } n_2) = K = K_b + K_v$

$$K = -\frac{C_b I_b \lambda_1 T}{2n_1} - \frac{C_b I_b \lambda_1 \theta T^2}{6n_1^2} + \frac{C_b I_b \lambda_1 \theta T^2}{3n_1^2} + \frac{C_b I_b \lambda_1 T}{n_1} + \frac{C_b I_b \lambda_1 \theta T^2}{2n_1^2} - \frac{C_b I_b \lambda_1 \theta T^2}{n_1^2}$$
$$+ C_b I_b \lambda_1 T - \frac{C_b I_b \lambda_1 \theta T^2}{2n_1^2} - \frac{C_b I_b \lambda_2 T}{2n_2} - \frac{C_b I_b \lambda_2 \theta T^2}{6n_2^2} + \frac{C_b I_b \lambda_2 \theta T^2}{3n_2^2} + \frac{C_b I_b \lambda_2 T}{n_2}$$
$$+ \frac{C_b I_b \lambda_2 \theta T^2}{2n_2^2} - \frac{C_b I_b \lambda_2 \theta T^2}{n_2^2} + C_b I_b \lambda_2 T - \frac{C_b I_b \lambda_2 \theta T^2}{2n_2^2} - \frac{C_b \theta \lambda_1 T}{2n_1} + C_b \theta \lambda_1 T$$
$$+ \frac{C_b \theta \lambda_1 T}{n_1} - \frac{C_b \theta \lambda_2 T}{2n_2} + C_b \theta \lambda_2 T + \frac{C_b \theta \lambda_2 T}{n_2} + \frac{n_1 A_{b1}}{T} + \frac{n_2 A_{b2}}{T}$$
$$+ C_v I_v \lambda_1 \theta T + C_v I_v \lambda_2 \theta T + \frac{C_v I_v \lambda_1 \theta T^2}{3} + \frac{C_v I_v \lambda_2 \theta T^2}{3} - C_v I_v \lambda_1 \theta T^2 - C_v I_v \lambda_2 \theta T^2$$
$$- \frac{C_v I_v \lambda_1 \theta T^2}{6} - \frac{C_v I_v \lambda_2 \theta T^2}{6} + \frac{C_v I_v \lambda_1 T}{2n_1} + \frac{C_v I_v \lambda_1 \theta T^2}{6n_1^2} - \frac{C_v I_v \lambda_1 \theta T^2}{3n_1^2} - \frac{C_v I_v \lambda_1 T}{n_1}$$

$$-\frac{C_v I_v \lambda_1 \theta T^2}{2n_1^2}+\frac{C_v I_v \lambda_1 \theta T^2}{n_1^2}-C_v I_v \lambda_1 T+\frac{C_v I_v \lambda_1 \theta T^2}{2n_1^2}+\frac{C_v I_v \lambda_2 T}{2n_2}+\frac{C_v I_v \lambda_2 \theta T^2}{6n_2^2}$$
$$-\frac{C_v I_v \lambda_2 \theta T^2}{3n_2^2}-\frac{C_v I_v \lambda_2 T}{n_2}-\frac{C_v I_v \lambda_2 \theta T^2}{2n_2^2}+\frac{C_v I_v \lambda_2 \theta T^2}{n_2^2}-C_v I_v \lambda_2 T+\frac{C_v I_v \lambda_2 \theta T^2}{2n_2^2}$$
$$+\frac{C_v \theta \lambda_1 T}{2n_1}-\frac{C_v \theta \lambda_1 T}{n_1}-C_v \theta \lambda_1 T+\frac{C_v \theta \lambda_2 T}{2n_2}-\frac{C_v \theta \lambda_2 T}{n_2}-C_v \theta \lambda_2 T+\frac{A_v}{T}----(33)$$

where K is a function of discrete variables n_1, n_2 and continuous variable T.

6. Parameter Estimation

6.1 Maximum liklihood estimation

The probability density function of the exponential distribution is given by,

$$f(x\,;\lambda) = \begin{cases} \lambda e^{-\lambda x} & if \quad x \geq 0 \\ 0 & if \quad x < 0 \end{cases} \text{---------(1)}$$

where λ is the parameter to be estimated. The MLE of λ given by $\frac{n}{\sum_{i=1}^{n} x_i}$

6.2 Bayes estimation

In this section, we consider the Baye's estimation for the parameter λ of exponential distribution assuming the conjugate of prior distribution for λ as two parameter Gamma distribution given as

$$f(\lambda/\alpha,\beta) = \begin{cases} \frac{\beta^\alpha}{\gamma(\alpha)} \lambda^{\alpha-1} e^{-\beta x} & ,\lambda \geq 0 \\ 0 & ,\lambda < 0 \end{cases} \qquad \alpha > 0, \beta > 0$$

The likelihood function is assumed as L (λ/x) and the posterior distribution is,

$$p(\lambda/x) \propto L(\lambda/x)\ f(\alpha,\beta)$$

$$p(\lambda/X) \propto \lambda^{n+\alpha-1}\ e^{-\lambda[\beta+\sum_{i=1}^{n} x_i]}$$

This follows Gamma distribution with parameter $\gamma(n+\tilde{\alpha}\ \ ,\tilde{\beta}+\sum_{i=1}^{n} x_i)$

The mean and variance are given by

Mean= $\frac{\alpha}{\beta} = \frac{n+\tilde{\alpha}}{\tilde{\beta}+\sum_{i=1}^{n} x_i}$ Variance= $\frac{\tilde{\alpha}}{\tilde{\beta}^2}$

7. Numerical simulation

To compare the different estimators of the parameters λ_1 and λ_2 of the exponential distribution, the risks under squared error loss of the estimates are considered. These estimators are obtained by maximum likelihood and Baye's methods under Expected risk. The MCMC procedure for Baye's estimation is as follows

(i) A sample of size n is then generated from the density of the exponential distribution, which is considered to be the informative sample.

(ii) The MLE and Baye's estimators are calculated with $\alpha = n+\tilde{\alpha}\ \ ,\beta = \tilde{\beta}+\sum_{i=1}^{n} x_i$

(iii) Steps (i) to (ii) are repeated N = 2000 times for different sample sizes and the risks under squared error loss of the estimates are computed by using:

Expected Risk ($\hat{\lambda}$)= $\frac{1}{N}\sum_{i=1}^{n}\left(\lambda_i-\hat{\lambda}\right)$ Where, $\hat{\lambda}_i$ is the estimate at the i[th] run

Assuming the value of λ_1 = 0.011, λ_2 = 0.012 the estimated value of $\hat{\lambda}_1$ and $\hat{\lambda}_2$ using MLE and Baye's along with Expected risk are given in Table 1 & 2.

Table 1 Parameter estimation of $\hat{\lambda}_1$ and expected risk

n	Criteria	$\alpha = n+\tilde{\alpha}\ \ ,\beta=\tilde{\beta}+\sum_{i=1}^{n}x_i$			
		MLE	**α=0.5, β=0**	**α=1, β=0.5**	**α=1.5, β=1**
10	Estimated value ER	0.0124 0.0000004	0.0122 0.0000008	0.0121 0.0000008	0.0121 0.0000008
25	Estimated value ER	0.0115 0.00000006	0.0114 0.00000006	0.0114 0.00000006	0.0114 0.00000006
50	Estimated value ER	0.0109 0.00000003	0.0108 0.00000003	0.0108 0.00000003	0.0108 0.00000003
75	Estimated value ER	0.0108 0.00000003	0.0108 0.00000003	0.0108 0.00000003	0.0108 0.00000003
100	Estimated value ER	0.0108 0.00000003	0.0108 0.00000003	0.0108 0.00000003	0.0108 0.00000003
125	Estimated value ER	0.0108 0.00000003	0.0108 0.00000003	0.0108 0.00000003	0.0108 0.00000003
150	Estimated value ER	0.0108 0.00000003	0.0108 0.00000003	0.0108 0.00000003	0.0108 0.00000003

Table 2 Parameter estimation of $\hat{\lambda}_2$ and expected risk

N	Criteria	$\alpha = n+\tilde{\alpha}\ \ ,\beta=\tilde{\beta}+\sum_{i=1}^{n}x_i$			
		MLE	**α=0.5, β=0**	**α=1, β=0.5**	**α=1.5, β=1**
10	Estimated value ER	0.0132 0.0000009	0.0131 0.0000008	0.0131 0.0000008	0.0130 0.0000008
25	Estimated value ER	0.0130 0.00000005	0.0128 0.00000005	0.0127 0.00000005	0.0127 0.00000005
50	Estimated value ER	0.0120 0.00000004	0.0119 0.00000004	0.0119 0.00000004	0.0119 0.00000004
75	Estimated value ER	0.0119 0.00000004	0.0119 0.00000004	0.0119 0.00000004	0.0119 0.00000004
100	Estimated value ER	0.0119 0.00000004	0.0119 0.00000004	0.0119 0.00000004	0.0119 0.00000004
125	Estimated value ER	0.0119 0.00000004	0.0119 0.00000004	0.0119 0.00000004	0.0119 0.00000004
150	Estimated value ER	0.0119 0.00000004	0.0119 0.00000004	0.0119 0.00000004	0.0119 0.00000004

It is seen that for small sample sizes the estimators under the Expected Loss function have smaller ER when choosing proper parameters α and β. But for larger sample sizes (n>50) , all the estimators have approximately same ER. The obtained results are demonstrated in Table 1 & 2. The estimated value of λ_1 is $\hat{\lambda}_1$ = 0.0108 and the estimated value of λ_2 is $\hat{\lambda}_2$ = 0.0119

8. Numerical illustration

To validate the proposed model, let us consider following example by considering two buyers (N=2) with their different demand.

Other parameter values considered in proper units for numerical analysis are

$[\lambda_1, \lambda_2, \theta, C_v, C_b, I_v, I_b, A_v, A_{b1}, A_{b2}]$ = [0.011, 0.012, 0.001, 10, 13, 0.15, 0.30, 5, 2, 2]

The numerical analysis of the integrated optimum model and independent policy lead to the following results

Table 3 Optimal solution of integrated and independent inventory policy

n_1	n_2	T	Buyer's Cost	Vendor's Cost	Total Cost
1	1	0.0299	133.7832	167.2225	301.0057
1	2	0.0406	147.7882	123.1508	270.939
1	3	0.0557	143.6331	89.7640	233.3971
1	4	0.0730	136.9948	68.4898	205.4846
1	5	0.0914	131.3015	54.7005	186.002
2	1	0.0446	134.5346	112.1055	246.6401
2	2	0.0374	213.9079	133.6882	347.5961
2	3	0.0389	257.0736	128.5331	385.6067
2^*	**4^***	**0.0447**	**111.855**	**268.4611**	**380.3161**
2	5	0.0528	265.157	94.6948	359.8518
3	1	0.0632	126.5898	79.1110	205.7008
3	2	0.0417	239.8126	119.9023	359.7149
3	3	0.0333	360.3638	150.1488	510.5126
3	4	0.0315	444.4476	158.7289	603.1765
3	5	0.0329	486.3255	151.9744	638.2999
4^*	**1^***	**0.0837**	**119.4841**	**59.7333**	**179.2174**
4	2	0.0500	240.0053	99.9979	340.0032
4	3	0.0336	416.6701	148.8082	565.4783
4	4	0.0262	610.6896	190.8387	801.5283
4	5	0.0230	782.611	217.3904	1000.0014

In table 3, the optimal solution is exhibited for integrated and independent inventory policy, then ordering policy is on (n_1 = 4, n_2 = 1) instead of (n_1 = 2, n_2 = 4). Optimum Total Cost for both integrated and independent policy is Rs.179.2174 and Rs.380.3161 respectively. The R-software is used and the outputs for an integrated policy shown in the above table. The buyer's cost increase when both the buyers and vendor agree to joint decision. In the integrated policy, vendor benefits Rs.208.7278 and buyer loses Rs. 7.6291. It is logical for the vendor to offer some incentive for the buyers to accept the integrated policy. To attract buyers, the vendor should be willing to offer some discount up to certain percentage of his extra benefit due to the integrated approach and due to this strategy long term relation is maintained. The integrated policy reduces the integrated total cost defined as

PETC = $\dfrac{K(n_1,n_2)-K(n_1^*,n_2^*)}{K(n_1,n_2)}$ by 2.0109%.

The Optimal cycle time with total cost is also shown graphically in Figure 2

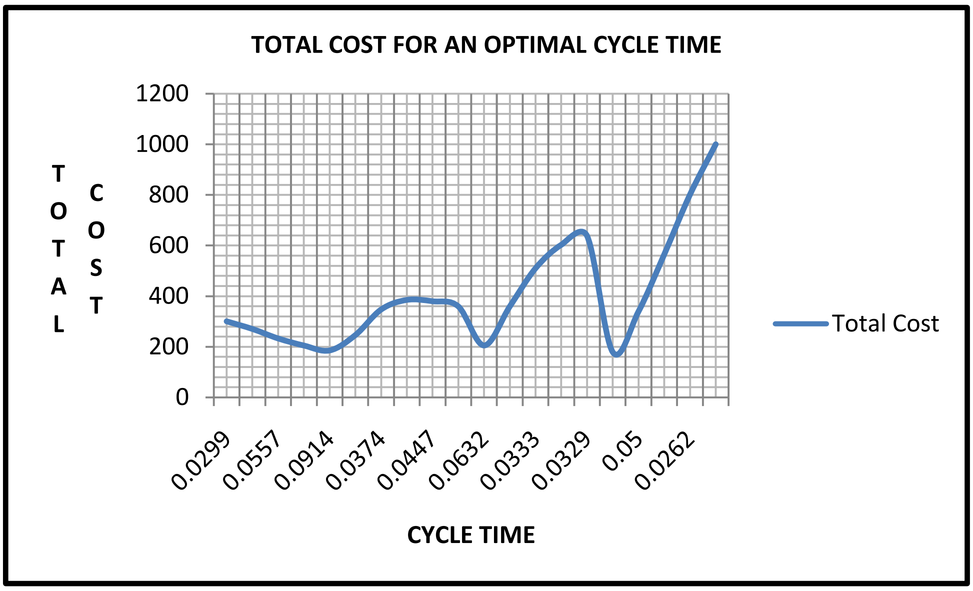

Figure 2 Total cost for an optimal cycle time

9. Sensitivity analysis

The effects of changes in the system parameter λ on the optimal ordering policy, cycle time T^* and TC^* are studied in the model. The sensitivity analysis is performed by changing each of the parameter by +25%, +10%, -10%, -25%, The results are shown in Table-4.

On the basis of the results in Table, as the changes of parameter increases in integrated and independent policy, the cycle time periods increases and total costs decreases which are shown graphically in figure 3 and 4.

Table 4 optimal solution of integrated and independent policy

Change in percentage	Independent		Integrated	
	Time	Total cost	Time	Total cost
+25	0.0664	256.0294	0.1133	132.402
+10	0.0528	287.9385	0.0949	158.0685
-10	0.0359	473.5397	0.0715	209.7948
-25	0.0229	712.0198	0.0531	282.4887

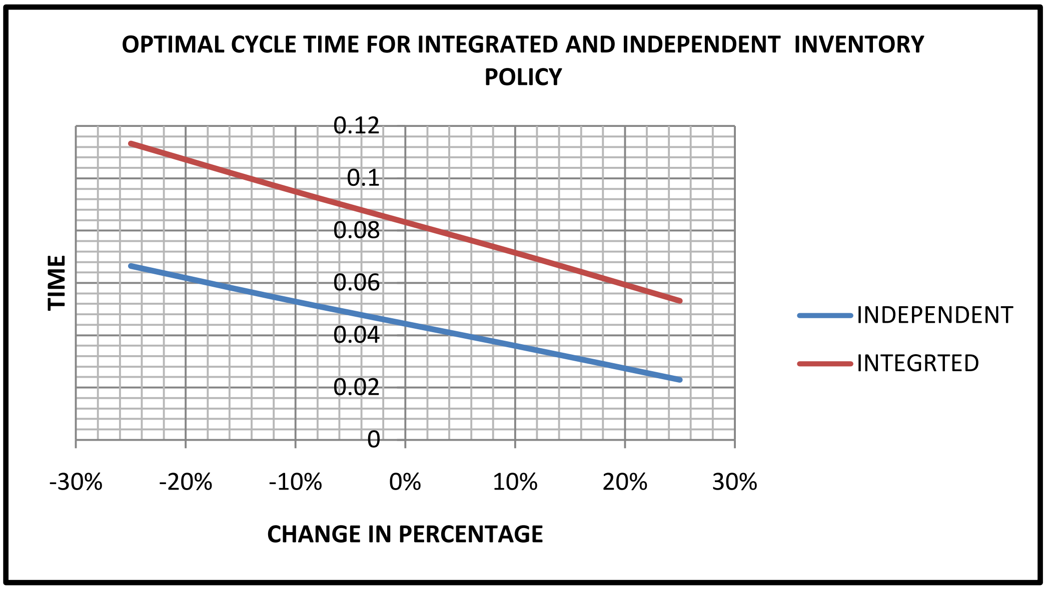

Figure 3 Optimal cycle time for integrated and independent inventory policy

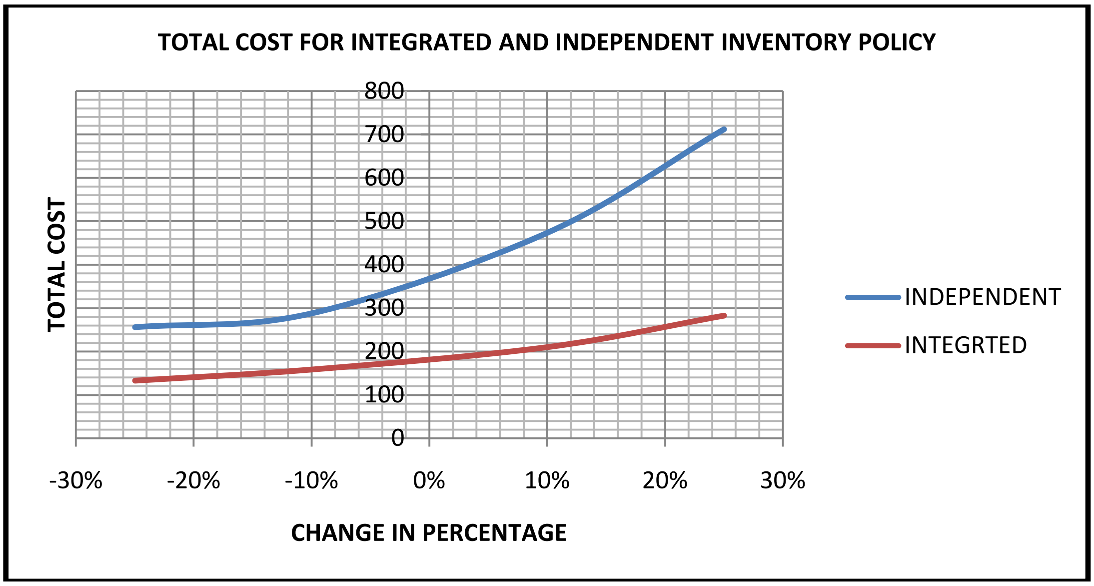

Figure 4 Total cost for integrated and independent inventory policy

10. Conclusion

A Markovian inventory integrated policy with inter-demand time as exponential distribution with a single-vendor and multiple-buyers integrated inventory model for deteriorating items is considered in this paper. The MLE and Baye's estimation are used to estimate the parameter and by using the estimated value a numerical illustration is given for different parametric values. The Optimal solution is exhibited for integrated and independent inventory policy, then ordering policy is on ($n_1 = 4$, $n_2 = 1$) instead of ($n_1 = 2$, $n_2 = 4$). Optimum total cost for both integrated and independent policy is Rs.179.2174 and Rs.380.3161 respectively. From the above model, the buyer's cost increase when both the buyers and vendor agree to joint decision. In the integrated policy, vendor benefits Rs.208.7278 and buyer loses Rs. 7.6291.

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